MATH MODEL OF RUNNING INDEX FOR EVALUATION AND CONTROL OF SPECIFIC EFFICIENCY AND TRACING THE FATIGUE LEVEL IN LONG-DISTANCE RUNNING

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ABSTRACT

The aim of this case study was to develop and verify a math model of running index (RI) for evaluation and control of specific efficiency and tracing the fatigue level in long distance running. The suggested RI was modelled on the basis of polynomial function between the running velocity and the change in heart rate frequency (V=f(HR)). The method was designed in such a way that the influence of denivelation of the terrain was removed when calculating RI. The research was done among 19 highly qualified athletes competing in long distance running events (16 men and 3 women), with a mean age of 28 years (SD±8), BMI 20 (SD±2), maximum oxygen consumption 67 ml/min/kg (SD ±5). The participants in the research were subjected to two lab and two terrain surveys. In a period of one week after the testing, RI was traced and calculated on the basis of the data taken from the major training loads. The model was verified only for the studied subjects - highly qualified competitors in long-distance running. RI was tested for validity through comparing the values obtained from the lab and terrain tests. The obtained results showed there was no significant difference between the values of RI measured during the lab testing and the terrain testing compared to the values of the velocity of ventilatory and lactate anaerobic threshold (V_{AnT}). The results from this research revealed that the suggested method for calculation of the running index is more accurate than the previous ones. This manuscript discusses the possible implications of this new method for measurement of RI. Future scientific research is needed to prove its efficiency and applicability in sport.

Keywords: *long-distance running, evaluation, control, fatigue level, running index*

INTRODUCTION

Sports achievement in long distance running is a function of numerous factors (Jeliazkov et al., 2020). The major factors related to the management of the training process are the conditioning level, the fatigue level, and the magnitude of the training load. The biological factors determining sports achievements are the maximum oxygen uptake (VO_2max), the anaerobic threshold (AnT), the energetic potential of the muscle (mainly the glycogen level in the muscle cell), and the economy of running (Neal, 2011; Midgley et al., 2007).

The measurement and tracing of these factors can be done through lab and terrain testing, with invasive and noninvasive methods. Some of these methods are expensive and their frequent use is not always applicable to the aims of the training process. Heart rate frequency during a physical load is an easily measured index which has a relation with the factors of sports achievement. The graph of the function between the heart rate and running velocity is approximately linear in the range of the anaerobic and anaerobic threshold. There is a fluctuation in the graph of this dependence at the beginning and the end before the above-mentioned thresholds (Ghosh, 2004; Kjertakov et al., 2016).

As far as we know, there are some surveys linked to the development of running index based on the linear – in a direct ratio to heart rate frequency and running velocity. The principle of calculation of this known index is based on the ratio between the heart rate frequency and running velocity. Its major use is for tracing the conditioning level of the athletes (Neikov, 2012; Vesterinen et al., 2014). The disadvantage of this method lies in the fact that for the different athletes, the graph of the function between heart rate and running velocity *V=f(HR)* can be with different position and shape in the coordination system. This is due to the genetic peculiarity of the cardiovascular system (Hui, Chan, 2006; Emig, Peltonen, 2020; Scharhag-Rosenberger et al., 2009). If the running index is calculated on the basis of the ratio between heart rate and running velocity (HR/Vrun), the measured values will not be comparable among different athletes. Also, in the regions where the graph function changes its linear shape, there will be a digression from the real values of the index.

In this manuscript, we present a running index (RI), which was modelled on the basis of the polynomial function *V=f(HR)* of second or higher degree, built individually for each competitor. This supposes an accuracy in the measurement of each point on the graph *V=f(HR)*. We also developed a method helping to remove the influence of denivelation of the terrain on the calculation of the running index. This makes it applicable in the real conditions of the training-competitive process.

To draw the graph *V=f(HR)*, we need the data about heart rate frequencies corresponding to the different running velocities. They can be measured with special testing or as a function of the database obtained in the training process itself. The developed math model of RI shows the digression of the values obtained from a certain physical load from the graph of the function *V=f(HR)* drawn on the basis of the initial data (from the testing). The obtained value is added to the initially measured threshold velocity (e.g., the velocity of level AnT or of a certain competitive velocity). RI has a numerical expression which is measured as the velocity (min/km, km/h, etc.) corresponding to a certain threshold physiological zone or competitive velocity.

The index can be applied for evaluation of the conditioning level, economy of running, fatigue level, and magnitude of the training load.

The increase in the conditioning level of an athlete is in relation to the increase of the hitting volume of the heart (Platonov, 2004; Astrand, Rodahl, 1986) and the increase in the efficiency of the parasympathetic heart control accompanied by a decrease in the heart rate frequency (Carter et al., 2003; Hellsten, Nyberg, 2015). Therefore, if the conditioning level of athletes increases, at the same values of heart rate frequency, they will be able to run at a higher velocity. With the increase in the conditioning level, the graph of the function will shift to the left without changing the degree of the angle concluded with the X-axis.

The increase in the economy of running is related to the increase in the efficiency of the organism to perform a certain amount of work with the same amount of used up energy (Saunders, et al., 2004). An indirect index of the used-up energy is the athlete’s heart rate frequency (Hiilloskorpi et al., 2003). The increase in the economy of a competitor leads to an increase in the running velocity while preserving the heart rate frequency. This is the reason why the graph of the function shifts to the right, similar to the increase of the conditioning level (Skinner et al., 2003).

The main reason for experiencing fatigue during the competitive-training process is the depletion of energy resources (glycogen) in muscle cells (Jeliazkov & Dasheva, 2010). This is the reason why they are replaced with higher threshold ones which, up to this moment, have not taken part in the muscle work (Wilmore, Costill, 1999; Dudley et al., 1982). This exchange is accompanied by an increased arousal of the central nervous system leading to an increase in the heart rate frequency while preserving the running velocity (Carter et al., 2003). Such an increase in the heart rate frequency can be provoked by tiredness of the heart muscle (EICF) (Dawson et al., 2003). We suppose that with loads leading to fatigue in a certain physiological zone, we will see shifting of parts of the graph *V=f(HR)*. With loads leading to fatigue in all physiological zones, we will see shifting of the whole graph to the left compared to its initial shape based on the data from the testing (Lambert et al., 1998) (Figure 1).

Day-to-day variation of heart rate frequency is well-known. It is a change in the heart rate frequency regardless of the known outer and inner reasons. The higher the intensity of the load, the lower the digression in the heart rate frequency is. The lowest digressions in the values of heart rate frequency can be observed at the intensity of the load between 85 and 90% (3 ± 1 bxmin) (Lamberts, Lambert, 2009). The standard error in measuring the submaximal heart rate is 1.1 – 1.4% (Lamberts et al., 2004).

In this case study, we introduce and verify a model of a running index for evaluation and control of conditioning level in the preparation of athletes competing in long-distance running events.

METHODOLOGY

The aim of this case study was to develop and verify a math method of running index for evaluation and control of specific efficiency in long-distance running.

In order to fulfil the aim of the research, we set the following *tasks*:

To develop a math model of running index for control and evaluation of specific efficiency in long-distance running;

To verify the math model of running index through supposition of its values obtained in lab and terrain testing.

Subject of the research is highly qualified competitors in long-distance running events (N = 19) (16 men and 3 women), mean age () 28 years (SD ± 8), BMI 20 (SD ± 2), maximum oxygen consumption 67 ml/min/kg (SD ± 5), mean velocity of level AnT 17.3 (SD ± 1.8). The sample is not representative and refers only to the subjects.

Research procedure

The participants in the research were subjected to two lab and two terrain testing. In a period of one week after the testing, we traced the major training loads of the participants in the case study. We recorded the mean values of the heart rate frequency, average running velocity, and denivelation of the terrain (in meters) where the load took place. We calculated the running index for each training and competitive load. The intensity of the training loads on the basis of which we calculated the RI has the following percentage distribution along physiological heart rate zones of load:

1st – zone – 45% (upper limit maximum fat oxidation (FatMax));

2nd – zone – 31% (upper limit second lactate threshold (AeT2));

3rd – zone – 15% (upper limit AnT);

4th – zone – 7% (upper limit velocity upon reaching maximum oxygen uptake

(VVO_{2max});

5th – zone – 3% (over VVO_{2max}).

The zones of the load were determined on the basis of the individual threshold zones (based on the heart rate frequency) measured with lab and lactate tests. The training loads took place on a firm terrain (asphalt, dirt road, tartan, or court stadium) and in relatively stable conditions of the environment (air humidity, temperature, etc.). The competitors got ready with their own training methods which were not changed for the purpose of this research.

The model was verified through statistical methods for comparison. The values of the index, calculated from the lab tests and the training loads were compared with the directly measured values from ventilatory AnT and lactate AnT (based on the level of the lactate – 4 mmol/l) ones, respectively. Also, the values of RI measured in the lab test were compared with the values measured through the old method for measuring the running index $RI_{HR/V}$.

Methods

Lab and terrain testing. In order to verify the model of the running index the following lab and terrain testing should be carried out:

1. Functional lab testing on treadmill including submaximal and maximal test. The tests were carried out within an interval of 20 minutes.

In the *submaximal test* the running velocity increases in a step-like way every 2 min with 1 km/h. The initial velocity is determined from the momentary conditioning of the athletes. The load at the end should not exceed the level of the anaerobic exchange so that the oxygen uptake be adequately used as an indicator for the used-up energy. The incline of the treadmill is 1% which to a great extent reflects the effort made when running on a flat terrain (Jones, Doust, 1996). The above-mentioned algorithm is preserved until one reaches a heart rate frequency equal to 70% of the maximal heart rate. Upon reaching the certain velocity there are three steps with different incline (1%, 4% and 7%) and duration of 4 min, and the running velocity is preserved. The physiological response of the organism is taken into consideration at different incline of the treadmill. The indicators used are heart rate frequency and oxygen uptake. The levels of lactate in blood are measured prior and after the test and at 3rd, 6th, and 9th minute. The aim of the test is to compare the energy losses of the athlete when running on different incline.

In the *maximal test* the running velocity increases in a step-like way every 2 min with 1.2 km/h, with incline of the treadmill of 1%. The initial velocity is determined on the basis of the results from the first test. The increase in the velocity is preserved until the athlete reaches the objective maximum. During the test the heart rate frequency, oxygen uptake, and the running velocity are tracked. The levels of lactate in blood are measured prior to the test and at the 3rd, 6th, and 9th minute after it. The aim of the test is to establish the ventilatory threshold velocities and the functional capacity of the athlete.

2. *The first terrain test* includes two runs of 1000 m. The first intercept is run on an incline ranging from 7 to 10%. The heart rate frequency corresponds to 2nd threshold zone. The second intercept is run on the same incline but in the opposite direction (descending). The heart rate frequency corresponds to 1st threshold zone. During the test we measure the average heart rate frequency, the denivelation in meters, and the running velocity achieved in the intercept. The aim of the test is to establish the change in the physiological response of the organism (measured through

the heart rate frequency) when running on a flat terrain, against an incline and down an incline.

3. *The second terrain test* is held within 3-5 days after the lab one and right after the first terrain test (20 min). In the period between the lab and terrain testing the training load is reduced. The aim is to achieve full recovery of the athlete's organism. The test includes running four intercepts with length 1600 m at a stadium. The running velocity during the intercepts is determined on the basis of the threshold velocities from the lab functional testing:

- 1st intercept – the velocity at *FatMax*;
- 2nd intercept – the velocity on level AeT_2 (Lactate = 2 mmol/l);
- 3rd intercept – the velocity on level *AnT*;
- 4th intercept – the velocity at VO_2max.

The lactate level is measured right before the test and after each intercept of 1600 m. During the test, the heart rate frequency and the running velocity are tracked in relation to the different lactate threshold zones. On the basis of this test, we build the function *V=f(HR)*.

Equipment and conditions

The lab spiroergometric functional tests were carried out in the scientific-research institute "Robert Koch" – NSA, on a treadmill "HP Cosmos Quasar med 4.0".

The terrain tests were carried out on a standard course and the following sports-technical means were used:

GPS watch for measuring the average indexes – denivelation, distance, and velocity (Polar Vantage V); heart rate meters (Polar H10 N Heart Rate Sensor);

Mini apparatus for measuring the level of lactate in blood (Arkray LactateProTM).

The terrain tests were carried out in relatively equal atmospheric conditions – wind speed up to 2 m/sec, air temperature between 10 and 25 degrees Celsius, and air humidity up to 85%.

Calculation of the running index

The heart rate frequencies during the lab functional test were significantly higher than the heart rate frequencies from the first terrain test compared to similar values at running velocity. We assume that the reason for the increase in the heart rate frequency was the specificities of the lab test, namely the used mask for gas analysis and running on a treadmill (where the colling effect is reduced to a minimum). Due to this reason, in order to build the graphs needed for calculating RI during the training experiment, we used the data from the terrain lactate test (Figure 1). The obtained equations describing the dependence are of the following kind and are a graph polynomic of third or higher degree:

$$V = aHR^3 + bHR^2 + cHR + d \quad (1)$$

$$HR = aV^3 + bV^2 + cV + d \quad (2)$$

Where:

a, b, c, d - function parameters;

HR – the change in the heart rate frequency;

V – the velocity corresponding to the heart rate.

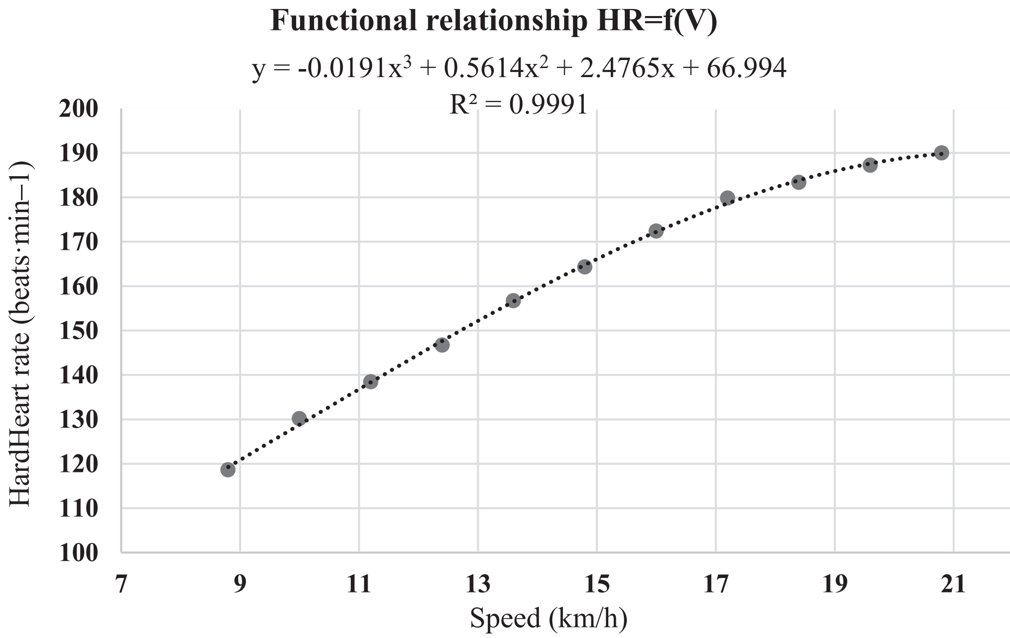

Figure 1. *Functional dependence between the heart rate and velocity measured during step-like maximal test on a treadmill*

If we assume that a certain training load is made after the graph of the function between the heart rate and velocity has been built, its value (graphically presented as p. A) (Figure 2) can lie on the curve (p. A) or be to the left or to the right from it (p. A_1).

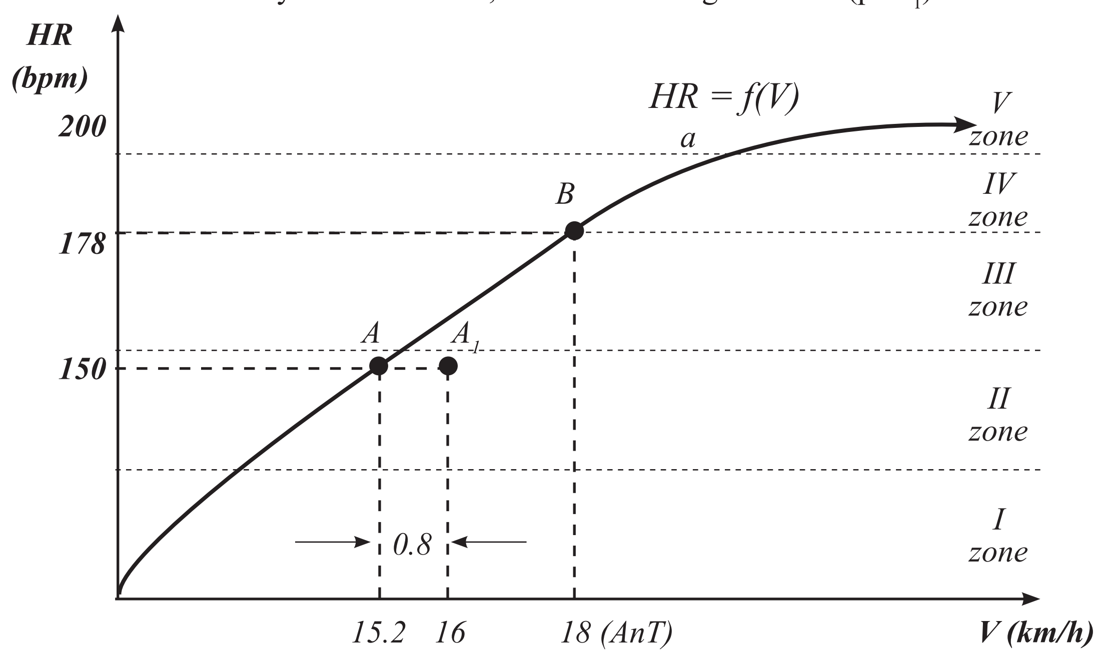

Figure 2. *Graph model of the modelled running index (RI)*

If p. A lies on the curve, with the increase in the running velocity, the athlete's heart rate frequency will also increase, following the dependence measured during the testing. The shifting of p. A upwards along the curve a will continue until p. A coincides with p. B, which corresponds to the velocity on level of AnT of a certain athlete. The viewed position of p. A in relation to the position of the curve a in short-term period (within the microcycle) means there is no sign of fatigue. If we trace it over a long-term period, if p. A remains unchanged, this means that there is no change in the conditioning level of the athlete.

If p. A does not lie on the curve a (in figure - p. A_1), the average heart rate frequency obtained after a certain training load is different from the heart rate frequency obtained in the testing while running at the same velocity. This measurement possibly has two geometrical projections – to the left or to the right of the curve a.

The reason for the shifting of p. A from the graph of the function can be due to several factors: a change in the level of conditioning of the athlete, a change in the level of fatigue, the surface of the terrain, the altitude the training load was held at, the air temperature and humidity, athlete's health condition, etc. (Achten, Jeukendrup, 2003; Lambert et al., 1998; Wilmore, Costill, 1999). If the conditions during the measurement are standard, the reasons for shifting in the position of p. A will be broken down into two – a change in the conditioning level of the athlete (Buchheit et al., 2010) and a change in the fatigue level (Boudet et al., 2004).

Turning the graph shifting of p. A_1 from the graph of the function (the curve a) into an empirical value is done with the following methods:

In the coordination system, certain heart rate frequency (150 beats/min) and running velocity (16 km/h) correspond to the position of p. A_1 (obtained after a particular training load) (Figure 2). At the same heart rate (150 beats/min), the curve a concurs with p. A corresponding to a different running speed (15.2 km/h). This change suggests that the athlete's velocity for a particular training load has increased or decreased in relation to the one measured during the testing. The difference in these velocities can be positive or negative and depends namely on the position of p. A_1 in relation to the curve *a*. The obtained value is added to the preliminarily established velocity of level AnT, their product is the value of RI. For instance, if the velocity is increased with 0.8 km/h compared to the data from the testing (16 km/h - 15.2 km/h), it is added to the velocity on level AnT (18 km/h) i.e., 18 km/h + 0.8 km/h =18.8 km/h (Figure 2). The obtained value is RI of the particular training load.

Removing the influence of denivelation of the terrain on the calculation of RI_n

The graph of the function between the magnitude of the incline (%) and the metabolic energy expenditure during running is relatively linear, ranging between ±10% of the incline. The loss of velocity during running while ascending is bigger than the benefit from identical descending (Minetti et al., 2002).

We obtained the following data from the second terrain test:

- heart rate frequency during running against an incline (HR_+)(beats/min);
- heart rate frequency during running along an incline (HR_-)(beats/min);
- velocity during running against an incline (V_+)(km/h);
- velocity during running along an incline (V_-)(km/h);
- denivelation ascend (D_+) (m);
- denivelation descend (*D*-) (m);
- running distance ascend (S_+) (m);
- running distance descend (S_-) (m).

The percentage of the incline (*D%*) is determined with the following formulas:

$$D\%_+ = \frac{D_+}{S_+} \quad (3)$$

$$D\%_- = \frac{D_-}{S_-} \quad (4)$$

From the function (1), we calculate the running velocity along a terrain without denivelation which corresponds to the heart rate during ascend (V_{HR+}) and consequently to the running velocity on a flat terrain which corresponds to the heart rate during descend (V_{HR}-). The running velocity against an incline is lower than that recorded at the same heart rate on a flat terrain. The running velocity along an incline (descending) is higher than the one recorded at the same heart rate on a flat terrain. Therefore, there is a loss during the ascend (V_{lost}) and a benefit during the descend (V_{gane}) at a constant physical load (measured through the heart rate frequency). These two quantities are calculated with the formulas (5) and (6) and depend on *D%*, where the test is carried out.

$$V_{lost} = V_{HR+} V_{+} \quad (5)$$

$$V_{gane} = V_{HR-} V_{-} \quad (6)$$

Therefore, there is a dependence between $D_{\%}$, V_{lost} and V_{gane}, which is expressed in the following way: the increase in $D_{\%}$ leads to an increase in V_{lost} and V_{gane}. The obtained equations describing the dependence are of the following kind:

$$V_{lost} = a_1 D_{\%+} \quad (7)$$

$$V_{gane} = a_2 D_{\%-} \quad (8)$$

Where a_1 and a_2 are parameters of the linear function.

The equation for calculating *RI* is of the following kind:

$$RI = ((\frac{V_{lost}\ Den_{+} - V_{gane}\ Den_{-}}{Den_{+} + Den_{-}} + V) - V_{HR}) + V_{AnT} \quad (9)$$

Where:

RI – running index, an indirect index about the change in velocity on anaerobic threshold level with denivelation included (km/h)

V_{lost} - the velocity lost under the influence of the incline upon ascend (7) ;

V_{gane} - the velocity gained under the influence of the incline upon descend (8);

Den_{+}- denivelation during the ascend (m);

Den_{-}- denivelation during the descend (m);

V – average velocity of the training (km/h);

V_{HR} - the velocity on a flat terrain which corresponds to the average heart rate during the training (km/h);

V_{Ant} - the velocity of *AnT* (km/h).

Statistical analysis

In the statistical processing of the data, we used the following statistical methods: variation analysis, tests for checking the normal distribution of the data - Kolmogorov-Smirnov and Shapiro Wilk, *t*-test Student for one sample. For the data processing, we used the software package for statistical analysis SPSS 25.

RESULTS

The preliminary analysis of the measured values from the tests and training loads showed the existence of a normal distribution, according to the applied tests of Shapiro-Wilk and Kolmogorov-Smirnov. That allowed us to use *t*-test Student for one sample and dispersion analysis ANOVA.

The effect size in testing the hypothesis calculated by Cohen's *d* for one sample *t*-test (Table 1 and 2). It is calculated as the difference between the mean of the data and the default value, all divided by the standard deviation of the data. Interpretation of effect sizes necessarily varies by discipline and the expectations of the experiment, but for this case study, the guidelines proposed by Cohen (1988) are: (0.2 – < 0.5 – Small Effect size; 0.5 – < 0.8 – Medium Effect size; ≥ 0.8 – Large Effect Size).

Table 1 presents the results from the individual *RI,* calculated for each step of the lab test until failure. With the help of *t*-test Student for one sample, the results from *RI,* calculated after the lab testing, were compared with the values of ventilatory *AnT*. *RI* was calculated on the basis of the function and $V_{gane/lost} f(D\%)$ built on the basis of the data from the lab tests.

The obtained results showed there was not a significant difference between the researched indexes at an error level $\alpha = .05$ (Table 1).

Table 1. *Comparison of the results about RI from the lab test with the ventilatory AnT*

Lab Test							
Athlete №	Number of measurements of *RI*	X mean *RI*	Std mean *RI*	Mean Difference	*AnT* (Ventilatory threshold)	Cohen's *d*	One-Sample *t*-test Sig
1.	9	18.00	0.24	0.002	18	0.00	.98
2.	8	18.00	0.16	0.000	18	0.00	.99
3.	8	16.78	0.18	-0.013	16.8	0.11	.85
4.	8	17.22	0.15	0.005	17.22	0.00	.93
5.	11	17.57	0.23	0.018	17.56	0.04	.80
6.	9	19.19	0.23	0.000	19.2	0.04	.99
7.	6	16.81	0.16	0.015	16.8	0.06	.82
8.	8	18.00	0.15	0.001	18	0.00	.99
9.	7	15.71	0.46	-0.021	15.74	0.07	.91
10.	9	16.79	0.15	-0.001	16.8	0.06	.98
11.	10	19.19	0.28	-0.001	19.2	0.04	.99
12.	7	18.00	0.15	0.000	18	0.00	.99
13.	6	15.60	0.09	0.004	15.6	0.00	.92
14.	7	18.00	0.07	0.000	18	0.00	.99
15.	10	19.56	0.28	-0.035	19.6	0.14	.70
16.	7	14.81	0.06	0.016	14.8	0.17	.50
17.	8	14.36	0.15	-0.038	14.4	0.26	.50
18.	6	13.43	0.14	0.003	13.43	0.00	.95
19.	7	17.47	0.03	-0.025	17.5	1.15	.05
Mean values	7.947	17.08	0.18	-0.004	17.09	0.11	.84

Table 2 shows the results from *RI* calculated on the basis of the data taken from the training loads of the athletes over a period of one week. *RI* was calculated on the basis of the function and $V_{gane/lost} = f(D\%)$ built on the basis of the data taken from the terrain tests. The results for *RI* obtained from the training sessions were compared with the lactate *AnT* with *t*-test Student for one sample. The obtained results showed there was not a significant difference between the researched indexes at error level $\alpha = .05$ (Table 2).

Table 2. *Comparison of the results for RI from the training loads with the lactate AnT*

Terrain Test							
Athlete №	Number of measurements of *RI*	X mean *RI*	Std mean *RI*	Mean Difference	*AnT* (Lactate test)	Cohen's *d*	One-Sample *t*-test Sig
1.	10	18.01	0.52	-0.191	18.2	0.37	.27
2.	24	18.61	0.50	0.014	18.6	0.03	.89
3.	10	17.43	0.60	-0.013	17.56	0.23	.49
4.	8	17.17	0.25	-0.047	17.22	0.19	.61
5.	16	17.10	0.41	0.293	17	0.25	.33
6.	16	19.44	0.41	0.165	19.5	0.14	.01
7.	12	16.53	0.36	-0.085	16.36	0.46	.14
8.	11	17.12	0.32	0.370	17.2	0.27	.40
9.	9	14.89	0.52	-0.140	14.52	0.72	.06
10.	9	17.42	0.44	-0.162	17.56	0.32	.37
11.	14	19.04	0.46	-0.071	19.2	0.36	.21
12.	24	17.83	0.44	0.036	17.9	0.16	.44
13.	16	15.44	0.35	-0.214	15.4	0.10	.69
14.	20	18.19	0.48	-0.199	18.4	0.44	.06
15.	16	19.31	0.41	-0.013	19.5	0.49	.07
17.	12	14.99	0.35	-0.085	15	0.04	.89
19.	11	17.12	0.52	-0.191	17.2	0.16	.40
Mean values	14	17.39	0.43	-0.031	17.431	0.28	.37

Table 3 presents the results from *RI* measured with the new method (*RI*) and *RI* measured with the old method ($RI_{HR/V}$). The results were compared with coefficient of variation. $RI_{HR/V}$ was calculated with the following formula (Vesterinen et al., 2014; Daniels, 1985) (https://fellrnr.com/wiki/Running_Economy):

$$RI_{HR/V} = \frac{(HRavgtren - (HRrest + 26))}{Vavgtren} \quad (10)$$

Where:
HRavgtren – average heart rate
Vavgtren – average speed
HRest – heart rate frequency at rest
26 – constant of the heart rate

The value of HR_{rest} used for calculating *RI* was the same for all athletes (50 beats/min). The constant of the heart rate determines the assumed change in heart rate frequency from lying to standing position (Hynynen et al., 2011). The value of the average heart rate during the load is reduced with the value of the heart rate frequency in standing position. Through this math action, significant digressions in the calculation are avoided since until reaching the value of the heart rate in standing position the athlete's velocity equals zero.

The mean values of the coefficient of variation (*V%)* for *RI* and $RI_{HR/V}$ were respectively equal to *0.010* (*SD* ± *0.004*) and *0.038* (*SD* ± *0.02*) (Table 3). The improved accuracy of measurement of the new method can be explained with the polynomial function, which describes more accurately the digressions from the linear dependence between the heart rate and the running velocity.

Table 3. *Comparison between the values of RI and $RI_{HR/V}$ measured during the lab test until failure*

Table of Comparison							
	RI				***$RI_{HR/V}$***		
Athlete №	Number of measurements of *RI*	X mean *RI*	Mean Std *RI*	V%	X mean $RI_{HR/V}$	Mean Std $RI_{HR/V}$	V%
1.	9	18.00	0.24	0.01	5.20	0.21	0.04
2.	8	18.00	0.16	0.01	5.25	0.17	0.03
3.	8	16.79	0.18	0.01	6.30	0.37	0.06
4.	8	17.23	0.15	0.01	6.25	0.20	0.03
5.	11	17.58	0.23	0.01	6.73	0.27	0.04
6.	9	19.20	0.23	0.01	4.83	0.27	0.06
7.	6	16.82	0.16	0.01	5.85	0.08	0.01
8.	8	18.00	0.15	0.01	5.16	0.08	0.02
9.	7	15.72	0.46	0.02	6.42	0.23	0.04
10.	9	16.80	0.15	0.01	5.39	0.13	0.02
11.	10	19.20	0.28	0.01	5.18	0.11	0.02
12.	7	18.00	0.15	0.01	6.18	0.21	0.03
13.	6	15.60	0.09	0.01	6.75	0.17	0.03
14.	7	18.00	0.07	0.00	6.29	0.31	0.05
15.	10	19.57	0.28	0.01	5.48	0.23	0.04
16.	7	14.82	0.06	0.00	6.53	0.37	0.06
17.	8	14.36	0.15	0.01	8.40	0.55	0.07
18.	6	13.43	0.14	0.01	8.75	0.56	0.06
19.	7	17.48	0.03	0.00	4.88	0.09	0.02
Mean values	7.95	17.08	0.18	0.01	6.10	0.24	0.04

Tables 4 and 5 present the results about the velocity loss V_{lost} upon overcoming +1% incline, measured on the basis of the change in oxygen uptake depending on the degree of the incline during the first lab test ($V_{lost\ 1\%V2max}$) and the velocity loss upon overcoming 1% incline, measured on the basis of the change in heart rate frequency depending on the degree of the incline during the first lab test ($V_{lost1\%HRlab}$ and $V_{lost1\%HRter}$).

$V_{lost\ 1\%V2max}$ was calculated with the following formula:

$$V_{lost\ 1\%VO2max} = \frac{((\frac{VO_{2avg3stage} - VO_{2avg1stage}}{VO_{2avg1stage}})Vr + Vr) - Vr}{D_{\%3stage}} VVO_{2differences} \qquad (11)$$

Where:
$VO_{2avg3stage}$ – average oxygen uptake measured during running on 3rd step (7%)
$VO_{2avg1stage}$ – average oxygen uptake measured during running on 1st step (1%)
V_r – running velocity (constant for all steps)
$D\%_{3stage}$ – percentage of the incline (third step – 7%)

The dependence between the percentage change of oxygen uptake and the percentage change in running velocity in the range between *AeT* and *AnT* is not completely in a direct ratio. The digression between the change in the two indexes can be calculated in the following way:

We find the percentage values in the change of velocity (V_{change}) and oxygen uptake ($VO_{2change}$) in the range between *AeT* and *AnT* with the following formulas:

$$V_{change} = \frac{V_{AnT} - V_{AnT}}{V_{AeT}} 100 \qquad (12)$$

$$VO_{2change} = \frac{VO_{2AnT} - VO_{2AnT}}{VO_{2AeT}} 100 \qquad (13)$$

We find the difference in percentage of the values of V_{change} and $VO_{2change}$ (14)

$$VVO_{2differences} = (V_{change} - VO_{2change}), \qquad (14)$$

The corresponding difference is multiplied with the obtained value for $V_{lost\ 1\%}VO_{2max}$.

To illustrate the results, we have presented distribution of the values of $V_{lost\ 1\%V2max}$, $V_{lost\ 1\%HR}$ and $V_{lost1\%HRtre}$ for the compared groups (Table 4).

After applying the statistical method Within Subject ANOVA the obtained results showed there was not a significant difference between $V_{lost\ 1\%V2max}$, $V_{lost\ 1\%HR}$ and $V_{lost1\%HRtre}$ at error level $\alpha = .05$ (*Sig.* = .735 > .05) (Table 5).

Table 4. *Distribution of the values of $V_{lost\ 1\%V2max}$, $V_{lost\ 1\%HR}$ and $V_{lost1\%HRtre}$ for the compared groups*

Descriptives								
Loss of Velocity at 1% Incline								
	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
$V_{lostHRter}$	19	0.534	0.132	0.029	0.474	0.596	0.26	0.71
$V_{lostVO2}$	19	0.536	0.173	0.040	0.453	0.620	0.19	0.77
$V_{lostHRlab}$	19	0.505	0.110	0.025	0.453	0.560	0.29	0.75
Total	57	0.525	0.140	0.018	0.489	0.562	0.19	0.77

Table 5. *Results from the dispersion analysis for the compared groups of values for $V_{lost1\%V2max}$, $V_{lost\ 1\%HR}$ and $V_{lost1\%HRtre}$*

ANOVA					
Loss of Velocity at 1% Incline					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.012	2	0.006	0.310	.735
Within Groups	1.049	54	0.019		
Total	1.061	56			

Table 6 presents the mean values and variability of *RI* measured during the first lab test on the second step with treadmill incline of 3% (RI_{2stage}), on the third step with incline 6% (RI_{3stage}) and the value of the velocity of the ventilatory *AnT* (V_{AnT}) (on this basis the measured indexes were made equal).

Table 6. *Mean values and variability of the results in the compared samples*

Group Statistics					
Stages		N	Mean	Std. Deviation	Std. Error Mean
RI	RI_{2stage}	19	17.44	2.03	0.46
	RI_{3stage}	19	17.22	1.82	0.42
	V_{AnT}	19	17.23	1.82	0.42

Table 7 presents the results from the dispersion analysis for the compared groups of values for RI_{2stage}, RI_{3stage} and V_{AnT}. The obtained results showed that there was no significant difference between the compared indicators at error level $\alpha = .05$ (*Sig. = .922*> .05) (Table 7).

Table 7. *Results from the dispersion analysis for the compared groups of values for RI_{2stage}, RI_{3stage} and V_{AnT}*

ANOVA					
$RI_{2stage,}$ RI_{3stage} u V_{AnT}					
	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	0.58	2	0.29	0.081	.922
Within Groups	193.10	54	3.58		
Total	193.68	56			

DISCUSSION AND CONCLUSIONS

A new method for calculation of *RI* was introduced for a direct tracing of the conditioning level and fatigue of athletes competing in long distance running events. The method is based on an individual approach when calculating the index. The obtained values for *RI* are numerical and are equal to a particular physiological threshold or competitive velocity. This improves its applicability when tracing and interpreting the values of the index. A method for measurement of the digression of the values of velocity during running on a terrain with denivelation was also introduced. The method could also be used when calculating *RI* with the known methods. The obtained results showed there was no significant difference between the values of *RI* measured during the lab testing and the terrain testing compared to the values of the velocity of ventilatory and lactate anaerobic threshold (VAnT). The results from this case study showed that the suggested method for calculation of the running index is more precise than the previous ones (Table 3). The model was verified only for the studied subjects - highly qualified competitors in long distance running. This advantage can be used for tracing the indexes which need greater accuracy in measurement.

During the training process the values of the heart rate frequency, running velocity, and denivelation of the terrain can be recorded in the form of a database. These data can be processed with a suitable software package (Fang et al., 2021; Emig, Peltonen, 2020; Fister et al., 2015) which can build the graph functions $V=(f)HR$ and $V_{gane/lost}=f(D\%)$ needed for the calculation of *RI*. The obtained graphs will be adapted and perfected over time depending on the individual changes in the physiological functions of the athlete. On the basis of the graph function $V=(f)HR$, we can find an inflexion point of the digression from the linear dependence, which is an indirect method for determining the level of the anaerobic threshold (Vachon et al., 1999). It is known that *AnT* is a major physiological factor for determining the training velocity and heart rate zones of load (Alexandrova, at al., 2018; Midgley et al., 2007; Kenney, et al., 2012). The changes in the values of the index would lead to a change in the zones of the load. This would improve the possibilities for finding the algorithm of the training process (Nie et al., 2021; Kumyaito et al., 2018; Hellard et al., 2006; Stoyanov, 2019).

We believe that the values of the running index should be traced dynamically during the training process. The tracking of *RI* over longer periods can be used for evaluation of conditioning level of the athletes. The change in the values of *RI* during a particular training load or during the microstructure of the training process can be a predictor for the fatigue level. We assume that the values of the index, as a fatigue indicator, will undergo some changes depending on the intensity, magnitude, and direction of the training load. This is due to the heterogeneous character of fatigue (Jeliazkov et al., 2020). For example, a training load with a high intensity (5^{th} zone) and a great magnitude will be a stress factor on the fast-twitch muscle fibers, while the same load will lead to stress of a low magnitude on slow-twitch muscle fibers (Dudley et al., 1982). This physiological peculiarity will be the reason why the fatigue from a physical load with a high intensity does not influence to a great extent the values of the running index measured during a load with a low intensity. This disadvantage can be avoided if the values obtained for *RI* are sorted on the basis of heart rate zones of load. Thus, the index will provide information about the fatigue level in the different physiological zones of the load. The values of *RI* can be used as a predictor for the advancing phase of supercompensation in a certain zone of the load.

Modern sports watches with GPS and heart rate meter provide the opportunity for recording the data measured during a certain load over different periods of time (usually for every second). Thus, the *RI* can be calculated and traced in dynamics during the training load. It is known that during loads at a constant velocity, the bigger the fatigue, the higher the heart rate (Boudet et al., 2004; Volkov et al., 2000; Monogarov, 1986; Platonov, 2004). We assume that the data from the running index obtained as a result of a greater training load that lead to a significant fatigue level will have the form of a descending point graph. Therefore, the reduced values of the running index can be explained with the increase in the level of fatigue in the athlete. Tracking the dynamic change in the values of the running index during running can find practical application for evaluation of the *fatigue level and the reached phase of su-*

percompensation during the training load.

RI is influenced by a number of additional factors such as air temperature, air humidity, surface of the terrain, altitude, level of mental fatigue, athlete's health condition, Day-to-day variation, etc. (Lamberts, Lambert, 2009; Van Cutsem et al., 2017; Casa et al., 2010; Achten, Jeukendrup, 2003; Kenney et al., 2012). The above-mentioned factors reduce the level of informativeness of *RI* about the indexes it aims to explain. This imposes the need for an additional analysis of the values of the index. The possible solution is to separate the values from the running index measured during the training loads in complicated conditions. If athletes train on identical terrains, the data from the running index can be accurately traced and compared with one another.

We have verified a new method of running index and discussed possible practical applications of the model. Future research is needed for confirming its efficiency and implication in sport.

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